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MODELLING TURBULENT TRANSPORT OF SOLIDS IN NON-NEWTONIAN CARRIER FLUIDS APPLICABLE TO TAILINGS DISPOSAL

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The increased volume of thickened tailings being sent to tailings storage facilities has created a need to predict what the transport characteristics of these non-Newtonian suspensions are, and what pumping capabilities are required. For very fine particle disposal, where the particle size is less than 20μm e.g. red mud, the material may be considered to behave as a homogeneous fluid and conventional non-Newtonian fluid pipeline design techniques can be successfully employed. However, in many thickened tailings, the size distribution is much broader than this, often containing sands that are added either before or after thickening. Where the concentration of the rheologically active particles are sufficiently high, such that the suspension may be considered to behave as a paste in laminar flow, then suitable non-Newtonian stratified flow models are used successfully to quantitatively model these flows. When the concentrations of the rheologically active particle are lower, such that the carrier fluid is a less viscous non-Newtonian fluid, turbulent flow is readily achieved. The frequent occurrence of this type of suspension in tailings and other streams creates the need to model turbulent flow of solids in non-Newtonian carriers. Many design methods for turbulent hydrotransport of solids in Newtonian fluids exist and the suitability of one of these methods is investigated in this paper for both industrial and laboratory suspensions.

KEY WORDS: non-Newtonian, suspension, turbulence, heterogeneous, tailings.

NOTATION

c_v, c_w	Volume and weight concentration	(-)
$c(y)$	Horizontal chord average concentration at vertical coordinate y.	(-)
C	Suspension's volume concentration = c_v	(-)
d, d_{max}	Particle diameter and maximum particle diameter	(m)
D	Pipe diameter	(-)
g	Gravitational constant	(ms^{-2})
k_1,k_2,k_3	Scaling constants	(-)
k	Consistency index	(Pa s^n)
n	Flow behaviour index	(-)
p	Pressure	(Pa)
S	Relative density	(-)
V_{50}	Velocity at which half of the solids are in suspension	(ms^{-1})
V_m	Mixture velocity	(ms^{-1})
V_{sm}	Maximum deposition velocity	(ms^{-1})
x	Axial distance	(m)
X	Particle size distribution fraction	(-)
y	Vertical distance from bottom of pipe	(-)

$\dot{\gamma}$	Shear rate	(s^{-1})
ρ	Density	(kg m^{-3})
τ, τ_y	Shear stress and yield stress	(Pa)

Subscripts

c	Carrier
e	Equivalent fluid
f	Fluid
h	Heterogeneous
s	Stratified or solid
w	Water

1 INTRODUCTION

The increased use of thickeners in tailings disposal has meant that many tailings, which previously could be considered simply as aqueous heterogeneous suspensions, now exhibit non-Newtonian characteristics. For fine and very viscous slurries the various modes of transport, described in the abstract, are understood and can be robustly designed. For less viscous slurries that can be transported in turbulent flow the situation is more complex and less well understood. Many designers believe that such suspensions will effectively behave as a homogeneous equivalent fluid. That is, rheological measurements made with the entire slurry, regardless of the particle size distribution, can be used to characterise the flow under turbulent flow, assuming that all of the suspension behaves as a single phase, i.e. an equivalent fluid. It is generally believed that common, ground mineral particles greater than 20 µm, or thereabouts, in size are too massive to contribute to the underlying carrier fluid rheology, and so must be conveyed as a coarse burden. Thickened tailings suspensions usually contain particles orders of magnitude larger than this. This is particularly true when sands are added to the tailings stream, either before or after thickening. Rheological measurements of such slurries are very difficult to achieve, with the coarser particles settling out in bench rheometers, or stratifying in pipelines used for rheological characterisation. Such behaviour is often recognised by practitioners, but determination of the extent of the stratification is difficult without specialised instrumentation, e.g. tomography rings on pipelines, and therefore, sometimes ignored. The resulting pseudo-rheological characterisation can be used to a limited extent, providing the particle sizes are small, but for slurries containing larger particles, e.g. sands, which are usually characterised using pilot plant or small-scale pipelines, the ability to use this whole slurry pseudo-rheology is restricted to use in very minor pipeline scale up.

Research in recent years has shown the particles moving through non-Newtonian fluids seem to behave in a very similar manner to those moving through Newtonian fluids, providing the viscosity of the fluid immediately next to the particle is used. This local viscosity is a function of the local shear rate, i.e. velocity profile, as well as that due to any shearing that the particle itself may produce. At the same time, mathematical modelling (e.g. Rudman & Blackburn 2012), and, to a very limited degree, experimental measurements of turbulent non-Newtonian fluids (e.g. Güzel et al 2009) has demonstrated that these fluids behave in a similar way to Newtonian fluids. Assuming that the suspension will behave as a homogeneous fluid is akin to using an equivalent fluid model for Newtonian suspensions, a model that is known to be very limited in its application and generally only for high Reynolds number flows, unlike those in non-Newtonian industrial flows. It is therefore reasonable to ask whether other models of suspension behaviour in Newtonian fluids might not be employed to predict the behaviour of non-Newtonian suspensions, once the non-Newtonian influences have been incorporated. It is this approach that is examined in this paper, where, rather than assuming that the suspension may be considered to behave homogeneously it is implicitly assumed that the suspension will behave heterogeneously and be modelled using an existing heterogeneous model. But first it is necessary to discuss some of the claims made in this introduction.

2 EVIDENCE OF UPPER SIZE LIMIT TO RHEOLOGICALLY ACTIVE PARTICLES

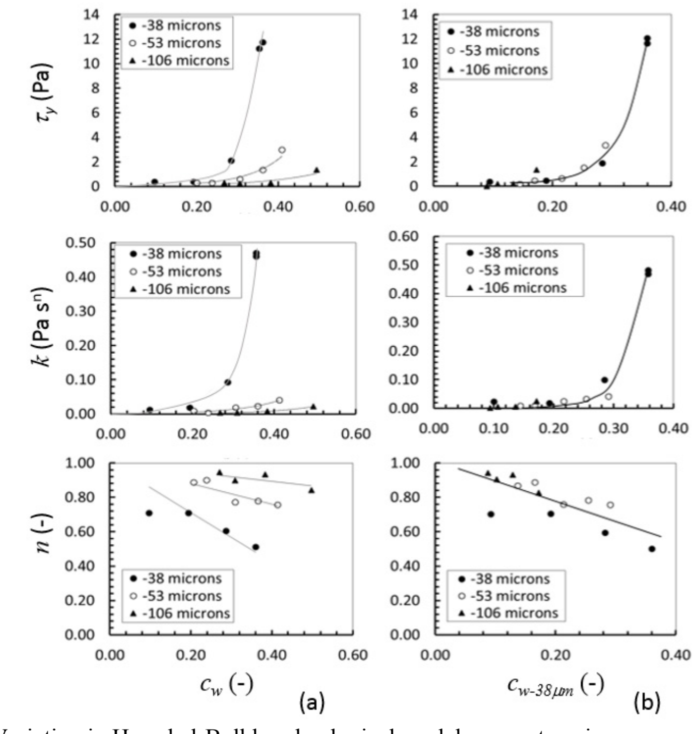

Figure 1Variation in Herschel-Bulkley rheological model parameters, i.e. $\tau = \tau_y + k\dot{\gamma}^n$, with concentration for three suspensions differing in top size (a) expressed as a function of total solids concentration, (b) expressed as a function of the concentration of -38µm particles.

For the suspension to behave as a homogeneous fluid all of the particles must take part in the production of the non-Newtonian fluid. However, while colloidal scientists would claim that the upper limit of the particle sizes that take part in the production of the non-Newtonian fluid is of order microns, there is evidence to suggest that the mineral suspensions, which enjoy low strength agglomeration of particles, are of the order of tens of microns, with perhaps 20µm being a practical upper limit. To investigate this some thickened tailings were scalped at three different top sizes and the variation in the rheological parameters with concentration examined (Figure 1).

When expressed as a function of the total solids content the rheological parameters for the three different top size suspensions produce different functional forms. If, however, for this material, the concentration of the suspensions is expressed in terms of the -38µm particle content, then the functional relationships collapses to a single form for each parameter. The implication is that particles in excess of 38µm do not contribute to the rheological behaviour of the slurry, but rather that it is governed by particles with a top size less than or equal to the 38µm size.

3 EVIDENCE OF HETEROGENEITY.

Laboratory experiments conducted with laminar flows have demonstrated that the coarse solids stratify to form a sliding bed (Pullum & Graham 2000, Talmon & Mastbergen 2004) and anecdotal evidence obtained in pilot plants has implied similar behaviour (Martinson et al 2013). For slurries typical of thickened tailings, cursory comparison of the particles' Stokes numbers based on characteristic particle settling velocities under shear and characteristic eddy velocities suggest that once a particle is engulfed by an eddy it will stay within that eddy, even when the turbulence levels are very low. This suggests that the coarser solids will be uniformly distributed throughout the pipe through the strong mixing action of the turbulent flow. Such behaviour would be synonymous to that of an equivalent fluid. Examination in the laboratory, however, shows that this is not the case and concentration profiles are seen within the pipe, even for relatively low concentrations of these coarser particles which might be typical of thickened tailings that contain sands.

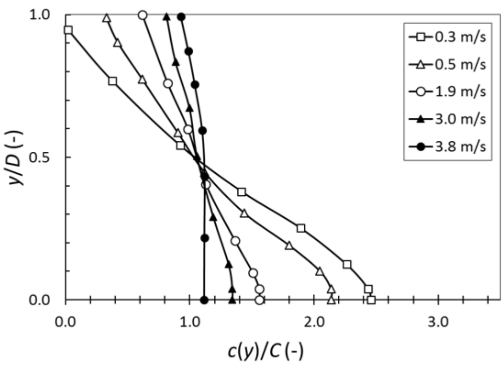

Figure 2 Normalised ERT concentration profiles for glass beads in a Carbopol suspension.

Figure 2 shows that even the turbulent concentration profiles (solid symbols) are not uniform across the pipe, increasing slightly as the solids approach the lower wall. Although these turbulent concentration profiles are reasonably uniform, it is believed that there is sufficient particle/wall interaction to increase the required transport pressure gradient above that obtained with an equivalent fluid.

4 COMPARISON OF THE TRANSPORT CHARACTERISTICS FOR 4 DIFFERENT FLOW MODELS

Figure 3 shows typical transport characteristics of a sand suspension conveyed in a non-Newtonian aqueous gel along with results various models.

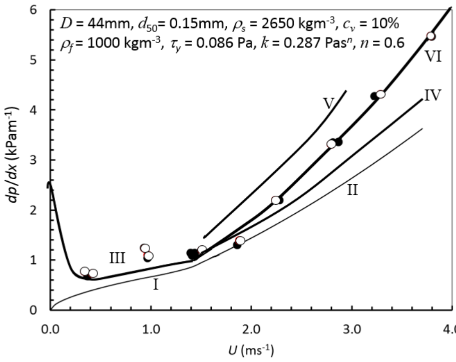

Figure 3 Transport characteristics of a 10% v/v sand suspension in Carbopol and various mathematical models: I, laminar analytical relationship, II, Wilson and Thomas, III non-Newtonian two layer model, IV equivalent fluid model, V, fully stratified non-Newtonian two layer model and VI three component model.

The underlying carrier fluid is well described using the Herschel Bulkley model, which in turn can be used to model the laminar flow curve using an analytical expression for the laminar flow (e.g. Shook and Roco 1991) (I), while the turbulent flow curve (II) can be modelled using the methods proposed by Wilson and Thomas (1985). As shown, (III), the laminar flow behaviour for the sand laden suspension is well captured using a non-Newtonian 2 layer model (Pullum et al 2004). If the suspension is assumed to behave as an equivalent fluid then the rheology of the Carbopol solution and the mixture density of the sand suspension can be used to produce curve (IV). This curve under-predicts the data implying that either the coarse sand solids do affect the rheology of the Carbopol, a theory not born out by experiment, or that there is some particle wall interaction, which increases the required pressure drop. Curve (V) is produced by a fully stratified two layer model, where all of the solids are conveyed as a sliding bed. Such behaviour is unlikely given the very small Stokes number that these particles have and the typical concentration profiles shown in Figure 2. Consequently it is not surprising that such a model over-predicts the required pressure drop. From these calculations it would seem reasonable to assume that the suspension is conveyed as a well suspended but heterogeneous suspension. Using a suitably modified version of a well-established heterogeneous model for Newtonian suspensions finally produces curve (VI) which is seen to capture the required pressure gradients very well. This model, and the modifications made are described in the next section. Note transitional flows, which are known to momentarily re-entrain solids, deposited in laminar flow, through the actions of turbulent puffs will not be considered here.

5 THREE COMPONENT MODEL

A broad size distribution heterogeneous model (Sellgren & Wilson 2007, Wilson et al 1990), developed from an original narrow size distribution model by Clift et al (1982) forms the basis of this three component model. In the original model the particle size distribution of the conveyed solids is split into four components as shown in Figure 4a, and the fractions of each of these components used to calculate the hydrotransport pressure gradient.

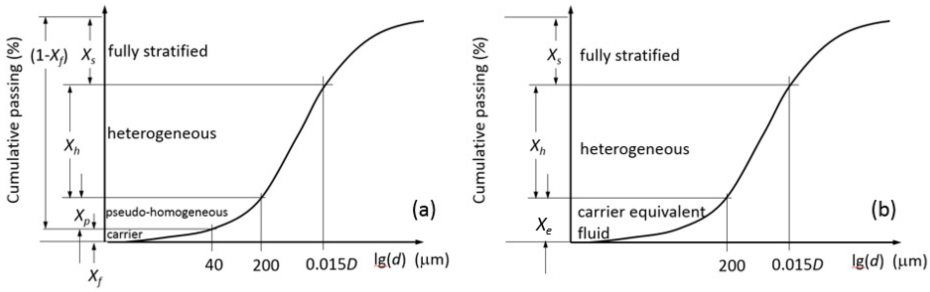

Figure 4 Subdivision of the particle size distribution for the original four component model (a) and the three component model (b).

The basis of the model is that well-established, semi-empirical formulas for each of the separate components shown in Figure 4a are used to compute the contribution of each component to the overall pressure drop. Starting with the finest particles, those less than 40µm, these particles are used to boost the Newtonian viscosity of the conveying fluid simply through their volumetric presence using formulations such as the Krieger and Dougherty relationship (Krieger & Dougherty 1959). This fine slurry is known as a carrier fluid in this case. Next, particles greater than those in the slurry, but less than 200µm are assumed to be uniformly suspended in the fine slurry to behave as a pseudo homogeneous equivalent fluid. This equivalent fluid is then used to convey particles less than 1.5% of the pipe diameter using the original stratified model of Clift et al. Finally particles larger than this, are conveyed as a sliding bed of solids submerged in the combined heterogeneous flow of the other three components.

The non-Newtonian version of this model is somewhat simpler, although the various non-Newtonian interactions need to be included. In the modified model the first two components, i.e. the carrier and pseudo-homogeneous components are combined into a single carrier fluid. This carrier fluid is the slurry that is typically measured in a benchtop viscometers, and comprises the base fluid and particles less than 40µm, i.e. the rheologically active ones and the -200μm particles. Without extensive detailed knowledge of non-Newtonian suspension behaviour the 200μm upper limit was selected as a reasonable value. It should be noted that other workers for example Shook et al (2002) advocate that the maximum size of particles that can be considered as part of the pseudo-homogeneous carrier fluid are those less than 74μm for low viscosity (e.g. water) fluids. In the present case, the use of a large particle (200μm) could be justified since the "background" viscosity of the fluid would be much larger for water and hence the fluid would be able to homogenously support larger than particles in turbulent flow. However, purely empirical constants have also been included in the resulting model to "fine tune" it. This carrier fluid is then used to convey the coarser particles in the same manner as the four component model, with the exception that settling velocities, where needed are based on the sheared viscosity of the suspension. The resulting subdivision of the particle size distribution is shown in Figure 4b.

The four component model assumes, somewhat tenuously, that the contribution from each fraction to the transport pressure gradient may be combined independently, i.e. $dp/dx_{tot} = dp/dx_c + dp/dx_p + dp/dx_h + dp/dx_s$. To varying degrees, each of the methods used to calculate the individual contributions contain empirically derived parameters originally developed from narrowly sized suspensions. The interaction between these fractions is not entirely negligible and so the validity of these parameters is not as rigorous and assertions about these are relaxed. Adding the complexity of non-Newtonian effects weakens these assertions even more. Based on the observations that (i) this model has been validated for a wide range of Newtonian systems in pipe sizes ranging from laboratory scale to industrial scale, and (ii) that these thickened tailings suspension appear to behave in a similar manner to their Newtonian counterparts, providing the local viscosity is used, it was decided to adapt this model to the following form

$$\frac{dp}{dx_{tot}} = k_1 \frac{dp}{dx_e} + k_2 \frac{dp}{dx_h} + k_2 \frac{dp}{dx_s} \qquad (1)$$

where the total pressure gradient comprises that due to the equivalent fluid based on the rheology of the carrier fluid described above, i.e. the carrier slurry, that due to the heterogeneous contribution from all particles between 200μm and $0.015D$, and the remaining (if any) coarse solids. Each of these components is also scaled by the constants $k1$, $k2$ & $k3$ to account for any enhanced non-Newtonian effects and these are determined empirically.

The contribution to the overall pressure gradients from each fraction is calculated in a similar way to the original four component model as follows:

<u>Equivalent fluid component</u>

This component is based on the underlying carrier fluids rheology and the density of the mixture of the fluid and the fraction of solids in this component, i.e. there is no excess contribution due to particle/wall interactions.

The density and relative density of the equivalent fluid is

$$\rho_e = \rho_c + X_e c_v (\rho_s - \rho_c), or$$
$$S_e = S_c + X_e c_v (S_s - S_c) \qquad (2)$$

where ρ_c and S_c is the actual carrier fluid concentration and relative density respectively, ρ_s is the solid's density and c_v is the total solids concentration. S_e, S_c and S_s are the relative density equivalents.

To calculate the pressure gradient, dp/dx_c for this non-Newtonian fluid it is necessary to use the fitted rheological model in conjunction with the Wilson and Thomas method (or equivalent) at the wall shear rate of interest. The resulting pressure gradient for the component is then given by

$$\frac{dp}{dx_e} = \frac{dp}{dx_c}\left[1 + (1 - 0.25X_e)X_e c_v(S_s - S_c)\right] \tag{3}$$

Heterogeneous fraction

The density of the heterogeneous slurry is

$$\begin{aligned}\rho_h &= \rho_c + (X_e + X_h)c_v(\rho_s - \rho_c), or\\ S_h &= S_c + (X_e + X_h)c_v(S_s - S_c)\end{aligned} \tag{4}$$

The contribution to the pressure gradient from the heterogeneous solids interaction is given by

$$\frac{dp}{dx_h} = \rho_w g B X_h c_v (S_s - S_e)\left(\frac{V_{50}}{V_m}\right) \tag{5}$$

where B is derived from a characteristic particle diameter d_h as follows

$$d_h = \frac{200\mu m + \min(0.015D, d_{max})}{2} \tag{6}$$

$$B = \begin{cases} 0.22 & d_h > 500\mu m \\ 0.22\dfrac{d_h - 200\mu m}{300\mu m} & otherwise \end{cases} \tag{7}$$

and

$$V_{50} = 3.93 {d_h}^{0.35}\left(\frac{S_s - S_e}{1.65}\right)^{0.45} \tag{8}$$

Stratified load fraction

The contribution from any sliding bed is approximated by

$$\frac{dp}{dx_s} = \rho_w g X_s c_v (S_s - S_h) 0.3\left(\frac{V_m}{0.55V_{sm}}\right)^{-0.25} \tag{9}$$

where V_m is the mixture velocity. The use of a deposition velocity in this last equation is problematic as the deposition velocity, V_{sm}, obtained by a non-Newtonian two layer model can be considerably larger than the Wilson type V_{sm} used in the original four layer model, especially for low values of n (Pullum et al 2004). To date however the various suspensions tested have not included solids large enough to fall into this fraction and so validation of this part of the model must wait.

The contribution from these three fractions is then summed to give the overall transport pressure gradient. The values for the three scaling values obtained so far are, $k_1 = 1.65$, $k_2 = 1$ and $k_3 = 1$. The increased effect of the pseudo-homogenous contribution appears reasonable as this contribution relies more heavily on the non-Newtonian character of the carrier fluid, and as noted above the limits to the pseudo-homogeneous regime is unknown at this stage.

This model has been compared with data obtained from three different suspension materials; a bimodal suspension of glass ballotini, with a density of 2450kgm^{-3}, conveyed in Carbopol and tested in a DN50 loop at the CVUT Civil Engineering laboratories in

Prague; a sand suspension, with a density of 2650kgm^{-3}, conveyed in Carbopol and tested in a DN40 loop at CSIRO laboratories in Melbourne; and a wide range of tailings material tested in DN100 and DN150 loops on site with similar densities (Coghill et al 2014). Representative particle size distributions of these materials are shown in Figure 5.

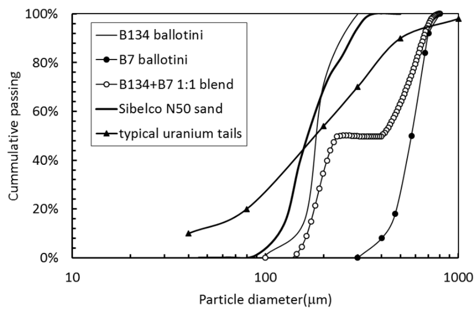

Figure 5 Particle size distributions of materials tested

It should be noted that all of these suspensions could be conveyed under both laminar and turbulent conditions and if minimum conveying velocities were observed, they were at values well below commercial interest. Pipeline blockage or the normal problems associated with Newtonian systems did not occur.

A parity plot comparing the predicted pressure gradients with those observed for these different materials and pipe sizes are shown in Figure 6.

A slight improvement between the observed and calculated pressure gradients can be made if the value of k_1 is allowed to float for the various suspensions, but the improvement is only marginal and without further data a functional form for k_1 cannot be established.

6 DISCUSSION AND CONCLUSIONS

An attempt has been made to use existing Newtonian methods to calculate the pressure gradient for turbulent suspension of fine particles typical of those used in thickened tailings disposal. Generally, such suspensions are modelled as homogeneous fluids, even though theoretical and empirical evidence suggests that such suspensions would be heterogeneous. The method chosen was a broad size distribution method that has had considerable success for Newtonian systems.

The apparent success of this method, after suitable modification, with the materials presented here indicate that the fundamental nature of suspension flows in Newtonian and non-Newtonian fluids are very similar, providing effects of shear are taken into account. To date, data from suspensions with particles sufficiently large to require minimum conveying velocities and sheared settling velocities to be included in the computations have not been available. It is the considered opinion of the authors that such suspensions will be adequately modelled using techniques similar to those presented in this paper, although minimum conveying velocities for these much coarser suspensions are still a research topic and needs to be developed.

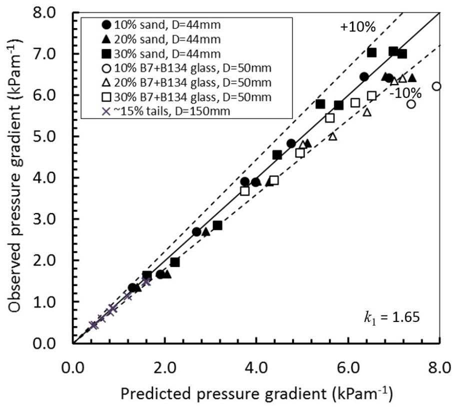

Figure 6 Parity plot comparing of the observed and predicted pressure gradients for a variety of suspensions and pipe sizes using the three component model and a fixed value of k_1.

7 ACKNOWLEDGEMENTS

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REFERENCES

1. Clift R, Wilson KC, Addie GR. 1982. *A Mechanistically-based method for scaling pipeline tests for settling slurrries.* Presented at International conference on the hydraulic tranpsort of solids in pipes - Hydrotransport 8., Johannesburg, South Africa
2. Coghill M, Jarvie N, Pullum L. 2014. *Operating Experience of a thickened tailings pilot plant.* Presented at 19th International conference on transport of the hydraulic transport of solids in pipes – Hydrotransport 19, Denver, USA
3. Güzel B, Burghelea T, Frigaard A, Martinez DM. 2009. Observation of laminar-turbulent transition of a yield stress fluid in Hagen-Poiseuille flow. *JFM* 627: 97-128
4. Kieger IM, Dougherty TJ. 1959. A mechanism for non-Newtonian flow in suspensions of rigid spheres. *Trans. Soc. Rheol.* 3
5. Martinson L, Martinson R, Cooke R. 2013. *Pipeloop tests at Codelco pilot plant.* Presented at Paste 2013, Belo Horizonte, Brazil
6. Pullum L, Graham L, Slatter P. 2004. *A non-Newtonian two layer model and its application to high density hydrotransport.* Presented at 16th International conference on transport of the hydraulic transport of solids in pipes – Hydrotransport 16, Santiago
7. Pullum L, Graham LJW. 2000. *The use of magnetic resonance imaging (MRI) to probe complex hybrid suspension flows.* Presented at 10th Transport and Sedimentation Conference, Wroclaw, Poland
8. Rudman M, Blackburn HM. 2012. *Turbulence Modification in Shear Thinning Fluids: Preliminary Results for Power Law Rheology.* Presented at 18th Australasian Fluid Mechanics Conference, Launceston, Australia
9. Sellgren A, Wilson KC. 2007. *Validation of a four-component pipeline friction-loss model.* Presented at 17th International conference on transport of the hydraulic transport of solids in pipes – Hydrotransport 17, Cape Town, Republic of South Africa
10. Pipeline Hydrotransport with Applications in the Oil Sand Industry. C.A. Shook, R.G. Gillies and R. S. Sanders. Saskatchewan Research Council Publication No. 11508-1E02., 2002.
11. Shook CA, Roco MC. 1991. Slurry Flow. Principles and Practice. Butterworth-Heinemann
12. Talmon A, Mastbergen D. 2004. *Solids transport by drilling fluids: Concentrated bentonite-sand-slurries.* Presented at 12th Transport and Sedimentation of Solid Particles, Prague, Czech Republic
13. Wilson KC, Clift R, Addie GR, Maffett J. 1990. Effect of broad particle grading on slurry stratification ratio and scale-up. *Pow.Tech.* 61: 165-72
14. Wilson KC, Thomas AD. 1985. A new analysis of the turbulent flow of non-Newtonian fluids. *Can. J. Chem. Eng.* 63: 539-46